

Prandtl numbers. Subscripts: tp, two-phase; b, boiling; con, convection; cr, critical; 0, single-phase; f, film; m, mixture; \*, reference value.

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#### THERMAL ENTRY ZONE OF A FILM FLOW

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A simplified solution for the thermal entry zone in the stabilization of a film is confirmed by experiments at  $Re \leq 450$  for boundary conditions of the first type.

In the calculation, design and use of film heat exchange equipment in industrial plants and cooling systems in energy engineering, the description and evaluation of the processes of flow and heat transfer under the conditions of film stabilization are important, as is the estimation of the optimum conditions of generating the film in the thermal entry zone, where a considerable danger exists of disrupting the continuity of the flow and forming dry patches on the heat exchange surfaces [1-10]. This problem has been considered in a number of papers, a detailed analysis of which has been given earlier [3-8]. During flow under nonisothermal conditions in the entry zone a transformation of the temperature profile occurs as well as the stabilization of the velocity profile. The relationship between the lengths of the hydrodynamic entry zone,  $x_H$ , and the thermal entry zone,  $x_T$ , is determined by the value of the Prandtl number,  $Pr$ . Thus, when  $Pr = 1$ ,  $x_H \approx x_T$ , but most often (when  $Pr > 1$ ),  $x_T > x_H$  [10].

A simplified analytical method has been proposed earlier [3] for calculating the velocity profiles and the lengths of the entry zones  $x_H$  for the laminar isothermal flow of wetting films on vertical surfaces, and this has been well confirmed experimentally in the zone  $Re \leq 450$ .

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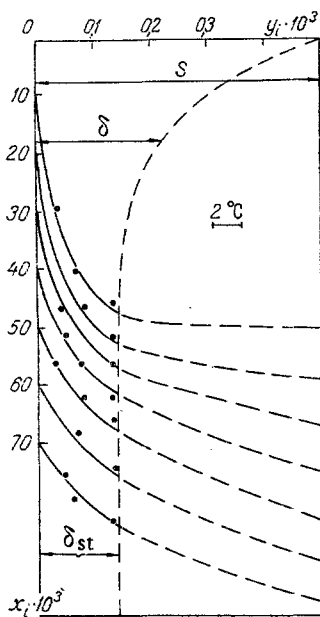


Fig. 1

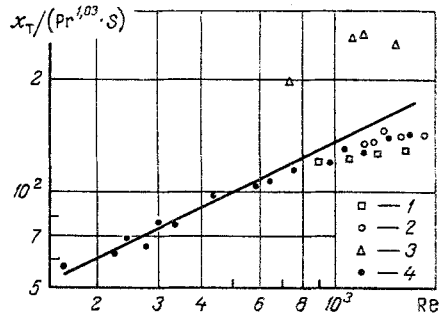


Fig. 2

Fig. 1. Local temperature profiles in the thermal entry zone of a water film at  $Re = 280$ ,  $\bar{t}_s = 88^\circ C$ ,  $t_{wall} = 104^\circ C = const$ , for various distances  $x_i$  in the direction of flow of the film.

Fig. 2. Comparison of data for  $x_T$ : straight line: calculation by Eq. (5); 1) data of Wilke [8]; 2) data of Struve [8]; 3) data of Radchenko et al. [8]; 4) data of the authors.

The same method can also be used for the simplified calculation of the temperature profiles and the lengths of the thermal entry zones  $x_T$  [4, 10]. Taking into account the main assumptions which have been made [4], the solution of the nonlinear dimensionless energy equation

$$w \frac{\partial t}{\partial x} = a^* \frac{\partial^2 t}{\partial y^2} \quad (1)$$

with respect to  $t$  was sought in the form of the power polynomial

$$t = f(x, y) = \kappa_0(y) + (x + \varepsilon)^{-1} \kappa_1(y) + (x + \varepsilon)^{-2} \kappa_2(y) + \dots, \quad (2)$$

where

$$w = \frac{\bar{w}_x}{\bar{w}_s}; \quad t = \frac{t_i}{\bar{t}_s}; \quad x = \frac{x_i}{S}; \quad a^* = \frac{a}{\bar{w}_s S}; \quad y = \frac{y_i}{S}. \quad (3)$$

The velocity profiles  $w = f(x, y)$  in the entry zone for hydrodynamic stabilization are calculated by the method of [3, 5]. The unknown functions  $\kappa_0, \kappa_1, \kappa_2, \dots$  of the transverse coordinate  $y$  are obtained as a result of solving a linear system of seven equations by the Gauss method of the successive elimination of unknowns [4, 5, 10]. Evaluation of the functions  $\kappa_7, \kappa_8, \dots$ , showed that they are very small and can be neglected.

By substituting the polynomial (2) and the value of  $w$  obtained earlier [3, 4] into Eq. (1) and carrying out the necessary rearrangements, a general expression is obtained for the temperature field in the entry zone:

$$t = [-A_0 y + A_1] + (x + \varepsilon)^{-1} \left[ -A_2 y - \kappa_0 \frac{y^2}{2} \right] + (x + \varepsilon)^{-2} \left[ -A_4 y - \frac{B_0}{2a^*} \kappa_1 y^2 \right] + (x + \varepsilon)^{-3} \left[ -A_6 y - \frac{B_0}{a^*} \kappa_2 y - \frac{B_1}{2a^*} \kappa_1 y^2 \right] + (x + \varepsilon)^{-4} \left[ -A_8 y - \frac{3B_0}{2a^*} \kappa_3 y^2 - \frac{B_1}{a^*} \kappa_2 y^2 - \frac{B_2}{2a^*} \kappa_1 y^2 \right] + \quad (4)$$

$$+ (x+\varepsilon)^{-5} \left[ -A_{10}y - \frac{2B_0}{a^*} \kappa_4 y^2 - \frac{3B_1}{2a^*} \kappa_3 y^2 - \frac{B_2}{a^*} \kappa_2 y^2 - \frac{B_3}{2a^*} \kappa_1 y^2 \right] + (x+\varepsilon)^{-6} \left[ -A_{12}y - \frac{5B_0}{2a^*} \kappa_5 y^2 - \frac{2B_1}{a^*} \kappa_4 y^2 - \frac{3B_2}{2a^*} \kappa_3 y^2 - \frac{B_3}{a^*} \kappa_2 y^2 - \frac{B_4}{2a^*} \kappa_1 y^2 \right],$$

where the constants  $B_0, B_1, B_2, B_3$  and  $B_4$  are already known from the calculation of the hydrodynamic stabilization zone [4, 5, 8], while the constants  $A_0, A_1, A_2$ , etc., are determined from the condition that the heat transfer process in the entry zone should be single-valued.

The main distinguishing feature of the solution (4) compared with the solution proposed earlier [4] is that it takes into account the initial velocity and temperature distributions in the stream (in the present case, at the exit from the slot of the distributing device).

The analytical solution of the problem [4] and its experimental confirmation were carried out for boundary conditions of the first type ( $t_{\text{wall}} = \text{const}$ ). The external wetting of stainless steel tubes of outside diameters 27.25 and 38.0 mm by distilled water and MK oil was investigated with temperatures at the inlet of 20–95.4°C, with  $S = 0.5$  mm,  $Re = 250$ –1430, for temperature differences between the wall and the liquid of 3–20°C. The local film temperatures were measured by an improved contact method using traversing microthermocouples [1, 8].

For each series of experiments the Reynolds number  $Re$  was held constant at some definite value, as were the other conditions corresponding to the calculation of the constants  $B_i$  and  $A_i$  and the wall temperature  $t_{\text{wall}}$ , and measurements were made of the local temperatures across the cross-section of the film at fixed values of its path length  $x_i$ . It should be noted that because of limitations in the contact method of temperature measurement (particularly at small values of the film thickness and under conditions of well-developed waviness at the larger values of  $Re$  and  $x_i$ ) it was often possible to carry out only 2–4 reliable temperature measurements over the cross-section of the film. For the particular values of the parameters in each experiment, the theoretical temperature profiles were calculated on a BESM-6 computer according to the expanded expression (4) using the DOS ASET package of applied programs [5].

In Fig. 1 the solid lines show the calculated temperature profiles in the film in the thermal entry zone with boundary conditions of the first type for various distances  $x_i$  in the direction of flow which were obtained by the analytical method proposed above [4, 10], while the points show the results of the corresponding measurements. The temperature profiles are shifted from the original profile (which in the present case is linear and independent of  $y_i$ ) at the exit from the slot of the distributing device to the fully developed profile at  $x_i = x_T$ . The temperature which is most characteristic for indicating the degree of thermal stabilization can be regarded as the temperature of the outer surface of the film,  $t_{\text{out}}$ , which was calculated from Eq. (4) for the case  $y_i = \delta_{\text{st}}$ . For establishing the end of the thermal entry zone, the deviation of  $t_{\text{out}}$  from the value determined by calculation for the given conditions was taken to be no more than  $\pm 1\%$  while in the experiments the accuracy of the relative temperature measurements over the film cross section was limited to  $\pm 0.02^\circ\text{C}$ .

The experimental data were in good agreement with the analytical relationship (4) over the range  $Re \leq 450$ , and for small path lengths of the film even for  $Re \leq 760$ . At increased rates of wetting, the mixing action of the waves increased and smoothed out the temperature changes across the transverse cross section of the film so that the calculated values became too high.

The investigations showed that the reduced length of the thermal entry zone cannot be described uniquely as a function of the Péclet number  $Pe$  alone, and it must therefore be sought as a function of the Reynolds number  $Re$  and the Prandtl number  $Pr$ .

For the wetting liquids mentioned above, the following relationship has been obtained for the length of the thermal entry zone under the conditions  $20 \leq Re \leq 988$  and  $1.85 \leq Pr \leq 133$  and with boundary conditions of the first type:

$$x_T = 4.67 Re^{0.49} Pr^{1.03} S = 2.06 Re^{0.82} Pr^{1.03} F_s^{-0.33} \delta_{\text{st}} \quad (5)$$

For these conditions, the length  $x_T$  fell in the range  $1.9 \cdot 10^{-2}$  to 10.55 m.

Figure 2 shows the values of  $x_T$  calculated by Eq. (5) as well as the experimental results. The approximating straight line describes the experimental data reasonably well even in the region of relatively large Reynolds numbers ( $Re \leq 950$ ), even though when the wave formation is

sufficiently well developed at the end of the hydrodynamic stabilization zone it is already impossible to neglect the transverse velocity components for  $Re > 150$  [3, 8]. Upon further increase of the Reynolds number the mixing action of the waves reduces the value of  $x_T$ .

#### NOTATION

$A_0, A_1, A_2, \dots; B_0, B_1, B_2, \dots$ , constants;  $a$ , temperature conductivity of the liquid,  $m^2/sec$ ;  $Fr_S = \bar{w}_S^2/gS$ , Froude number for the flow in the slot;  $g$ , accelerating force of gravity,  $m/sec^2$ ;  $Pe = r\Gamma_V/a$ , Peclet number for the film flow;  $Pr = \nu/a$ , Prandtl number;  $Re = 4\Gamma_V/\nu$ , film Reynolds number;  $S$ , slot width of distributing device,  $m$ ;  $t_{out}$ , temperature of outer surface of film,  $^{\circ}C$ ;  $t_{wall}$ , temperature of wetted surface of wall,  $^{\circ}C$ ;  $t_S$ , liquid temperature at exit from distributing slot,  $^{\circ}C$ ;  $t_i$ , local film temperature,  $^{\circ}C$ ;  $w_x$ , local velocity of the film flow along the  $x$  axis,  $m/sec$ ;  $\bar{w}_S$ , mean velocity of the liquid at the exit from the distributing slot,  $m/sec$ ;  $x_H, x_T$ , lengths of the hydrodynamic and thermal entry zones, respectively,  $m$ ;  $x_i, y_i$ , longitudinal (vertical) and transverse (horizontal) flow coordinates,  $m$ ;  $\Gamma_V$ , volumetric irrigation density,  $m^2/sec$ ;  $\delta$ , local film thickness,  $m$ ;  $\delta_{st}$ , film thickness in the hydrodynamically steady-state flow regime,  $m$ ;  $\epsilon$ , constant;  $\kappa_0, \kappa_1, \kappa_2, \dots$ , functions of the transverse coordinate  $y$ ;  $\nu$ , kinematic viscosity of liquid,  $m^2/sec$ .

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